GCE Examinations Advanced Subsidiary / Advanced Level

Statistics Module S3

Paper C MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



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S 3	Paper	C –	Marking	Guide
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1.	(<i>a</i>) 72, 65, 36, 61, 12, 17	M1 A2
	(b) e.g. advantage – avoids bias disadvantage – time consuming	B1 B1 (5)
2.	(a) $\overline{T} \sim N(28.5, \frac{7.2^2}{8}) = \sim N(28.5, 6.48)$	M1 A1
	(b) $P(25 < \overline{T} < 30) = P(\frac{25 - 28.5}{\sqrt{6.48}} < Z < \frac{30 - 28.5}{\sqrt{6.48}})$	M1 A1
	= P(-1.37 < Z < 0.59) = 0.7224 - (1 - 0.9147) = 0.637	M1 A1 (6)
3.	(a) $E(X) = (2 \times 0.05) + (4 \times 0.15) + (7 \times 0.3) + (k \times 0.5)$ = 2.8 + 0.5k	M1 A1
	(b) $E(2\overline{X} - 5) = 2(2.8 + 0.5k) - 5 = k + 0.6$ \therefore bias = 0.6	M1 M1 A1
	(c) unbiased est. of $k = 2\overline{X} - 5.6 = (2 \times 8.34) - 5.6 = 11.08$	M1 A1 (7)
4.	let <i>T</i> = total mass of waste $\therefore T \sim N(8 \times 6.8 + 3 \times 3.2, 8 \times 1.5^2 + 3 \times 0.6^2) = \sim N(64, 19.08)$ $P(T > 70) = P(Z > \frac{70 - 64}{\sqrt{19.08}})$ $= P(Z > 1.37) = 1 - 0.9147 = 0.0853$	M2 A2 M1 M1 A1 (7)
5.	H ₀ : $\mu_A = \mu_N$ H ₁ : $\mu_A < \mu_N$ 5% level ∴ C.R. is $z < 1.6449$ test statistic = $\frac{32.8-35.1}{\sqrt{\frac{4.6^2}{50} + \frac{8.0^2}{190}}} = 2.6382$ in C.R. ∴ reject H ₀ there is evidence that those in school teams complete task quicker	B1 B1 M2 A2 M1 A1 (8)
6.	expected freq. Highfield/English = $\frac{80\times46}{120}$ = 30.67 giving expected freqs 30.67 15.33 49.33 24.67 H ₀ : no difference in proportions at the two schools	M1 A1 M1 A1
	H ₁ : there is a difference in proportions at the two schools $O = E = (O - E) = \frac{(O - E)^2}{E}$ 32 30.67 1.33 0.0577 14 15.33 [−] 1.33 0.1154 48 49.33 [−] 1.33 0.0359 26 24.67 1.33 0.0717 $\therefore \Sigma \frac{(O - E)^2}{E} = 0.2807$ $v = 1, \gamma^2 \operatorname{crit}(10\%) = 2.705$	B1 M1 A2 M1 A1
	$0.2807 < 2.705$ \therefore not significant there is no evidence of a difference in proportions at the two schools	A1 (11)

	$\therefore \Sigma \frac{(O-E)^2}{E}$ $v = 4 - 2 = 2$ $1.219 < 5.99$ $Po(1.2) \text{ is a } $	2, $\chi^2_{\text{crit}}(5\%)$ 1 \therefore do no suitable mo) = 5.991 ot reject H ₀ odel		M1 A1	(20)
	$\therefore \Sigma \frac{(O-E)^2}{E} = 2$ $v = 4 - 2 = 2$ $1.219 < 5.99$	$2, \chi^2_{\text{crit}}(5\%)$	= 5.991		M1	
	$\therefore \Sigma \frac{(O-E)^2}{E}$	- 1.217				
		- 1 219			M1 A1	
	11	9.64	1.36	0.1919		
	14	17.35	-3.35	0.6468		
	23 32	24.10 28.91	3.09	0.0302		
	0	E 24.10	(U - E)			
	combining g	roups ≥ 3		$(O-E)^2$	IVI I	
	∴ exp. freqs	M1 A1				
	\times 80 to give					
	$P(4) = \frac{1.2^4 e^{-4}}{4 \times 3 \times 3}$					
	$P(3) = \frac{1.2^3 e^{-3}}{3 \times 2}$	$\frac{1.2}{1.2} = 0.086$	7		M1 A2	
	$P(2) = \frac{1.2^2 e^{-2}}{2}$	= 0.216	9			
	$P(1) = 1.2e^{-1}$	$^{1.2} = 0.3614$	4			
	$P(0) = e^{-1.2} =$	= 0.3012	inode	•	21	
(<i>d</i>)	$H_0: Po(1.2)$ $H_1: Po(1.2)$	B1				
(0)			10			
(c)	varianco ~ "	B 1				
	$\hat{\sigma}^2 = s^2 = \frac{3}{2}$	M1 A1				
	$\Sigma f x^2 = 32 + 32$, 56 + 72 + 4	48 = 208		M1	
(b)	$\hat{\mu} = \overline{x} = \frac{\sum x}{\nabla}$	$\frac{fx}{f} = \frac{96}{80} =$	1.2		M1 A1	
1	near fut	ire given le	ong half-life	e) so seems suitable	B3	
<i>(a)</i>	e.g. particles	are emitte	d singly, at	random and at a constant rate (for		
	should f	nd vice versa	B1	(11)		
()	e.g. it seems	ness of those with a given rest pulse	ы			
(a)	vomiables ne	ad to he ici		live distails stad	D1	
	-0.8628 < -(there is evid	0.5155 ∴ s ence that n	significant	lower rest pulse are fitter	A1	
	n = 20, 1% 1	evel \therefore C.	R. is $r < -0$.5155	M1 A1	
(b)	$H_0: \rho = 0$	$H_1: \rho < 0$			B1	
	$r = \frac{-2858}{\sqrt{1783.2 \times 62}}$	$\frac{.8}{156.95} = 0$.8628		M1 A1	
	$S_{pt} = 27188$	$\frac{1176\times511}{20}$	= 2858.8		M1	
	$S_{tt} = 19213$ -	$-\frac{511^2}{20} = 6$	156.95		M1	
(<i>a</i>)	$S_{pp} = 70932$	MI				
	 (a) (b) (c) (d) 	(a) $S_{pp} = 70932$ $S_{tt} = 19213 - S_{pt} = 27188 + r = \frac{-2858}{\sqrt{1783.2\times61}}$ (b) $H_0: \rho = 0$ n = 20, 1% 1 -0.8628 < -0 there is evide (c) variables near e.g. it seems should f (a) e.g. particles near futur (b) $\hat{\mu} = \overline{x} = \sum_{x} \sum_{x} \sum_{x} fx^2 = 32 + 32 + 32 + 32 + 32 + 32 + 32 + 3$	(a) $S_{pp} = 70932 - \frac{1176^2}{20} = S_{tt} = 19213 - \frac{511^2}{20} = 6$ $S_{pt} = 27188 - \frac{1176\times511}{20}$ $r = \frac{-2858.8}{\sqrt{1783.2\times6156.95}} = ^{-0}$ (b) $H_0: \rho = 0$ $H_1: \rho < 0$ $n = 20, 1\%$ level \therefore C. $-0.8628 < ^{-0.5155} \therefore s$ there is evidence that p (c) variables need to be joid e.g. it seems reasonable should follow a nor (a) e.g. particles are emitted near future given be (b) $\hat{\mu} = \overline{x} = \frac{\sum fx}{\sum f} = \frac{96}{80} = \sum fx^2 = 32 + 56 + 72 + 423 - 633 - 123 - 1$	(a) $S_{pp} = 70932 - \frac{1176^2}{20} = 1783.2$ $S_{u} = 19213 - \frac{511^2}{20} = 6156.95$ $S_{pt} = 27188 - \frac{1176\times511}{20} = -2858.8$ $r = \frac{-2858.8}{\sqrt{1783.2\times6156.95}} = -0.8628$ (b) H ₀ : $\rho = 0$ H ₁ : $\rho < 0$ $n = 20, 1\%$ level \therefore C.R. is $r < 0$ $-0.8628 < -0.5155$ \therefore significant there is evidence that people with (c) variables need to be jointly normal e.g. it seems reasonable that the fit should follow a normal dist. at (a) e.g. particles are emitted singly, at near future given long half-life (b) $\hat{\mu} = \overline{x} = \frac{\sum fx}{\sum f} = \frac{96}{80} = 1.2$ $\sum fx^2 = 32 + 56 + 72 + 48 = 208$ $\hat{\sigma}^2 = s^2 = \frac{80}{79} (\frac{208}{80} - 1.2^2) = 1.17$ (c) variance ≈ mean as would be expected (d) H ₀ : Po(1.2) is a suitable model H ₁ : Po(1.2) is not a suitable model H ₁ : Po(1.2) is not a suitable model H ₁ : Po(1.2) is not a suitable model H ₁ : $P(2) = \frac{1.2^2 e^{-1.2}}{3x^2} = 0.2169$ P(3) = $\frac{1.2^3 e^{-1.2}}{3x^2} = 0.0260$ × 80 to give exp. freqs then freq 6 \therefore exp. freqs are 24.10, 28.91, 17 combining groups ≥ 3 O E ($O - E$) 23 24.10 - 1.1 32 28.91 3.09 14 17.35 - 3.35 11 9.64 1.36	(a) $S_{pp} = 70932 - \frac{1176^2}{20} = 1783.2$ $S_n = 19213 - \frac{51!}{20} = 6156.95$ $S_{pr} = 27188 - \frac{1176(511)}{20} = -2858.8$ $r = \frac{-2858.8}{\sqrt{1783.2.6156.95}} = -0.8628$ (b) H ₀ : $\rho = 0$ H ₁ : $\rho < 0$ $n = 20$, 1% level \therefore C.R. is $r < -0.5155$ $-0.8628 < -0.5155$ \therefore significant there is evidence that people with lower rest pulse are fitter (c) variables need to be jointly normally distributed e.g. it seems reasonable that the fitness of those with a given rest pulse should follow a normal dist. and vice versa (a) e.g. particles are emitted singly, at random and at a constant rate (for near future given long half-life) so seems suitable (b) $\hat{\mu} = \overline{x} = \sum_{T} \frac{f_n}{2f} = \frac{96}{80} = 1.2$ $\Sigma f x^2 = 32 + 56 + 72 + 48 = 208$ $\hat{\sigma}^2 = s^2 = \frac{80}{79} (\frac{208}{80} - 1.2^2) = 1.17$ (c) variance ≈ mean as would be expected with a Poisson distribution (d) H ₀ : Po(1.2) is a suitable model H ₁ : Po(1.2) is not a suitable model P(0) = $e^{-1.2} = 0.3012$ P(1) = $1.2e^{-1.2} = 0.3614$ P(2) = $\frac{12^3e^{-1.2}}{2} = 0.0260$ × 80 to give exp. freqs then freq of ≥ 5 = (80 - sum of others) \therefore exp. freqs are 24.10, 28.91, 17.35, 6.94, 2.08, 0.62 combining groups ≥ 3 O E ($O - E$) $\frac{(O - E)^2}{E}$ 23 24.10 - 1.1 0.0502 32 28.91 3.09 0.3303 14 17.35 - 7.355 0.6468 11 9.64 1.36 0.1919	(a) $S_{pp} = 70932 - \frac{1176^2}{20} = 6156.95$ M1 $S_{n} = 19213 - \frac{511^2}{20} = 6156.95$ M1 $S_{pr} = 27188 - \frac{11766511}{20} = -2858.8$ M1 $r = \frac{-2858.8}{\sqrt{1783.3615695}} = -0.8628$ M1 A1 (b) H ₀ : $p = 0$ H ₁ : $p < 0$ B1 $n = 20$, 1% level C.R. is $r < -0.5155$ M1 A1 (c) variables need to be jointly normally distributed B1 e.g. it seems reasonable that the fitness of those with a given rest pulse should follow a normal dist. and vice versa B1 (a) e.g. particles are emitted singly, at random and at a constant rate (for near future given long half-life) so seems suitable B3 (b) $\hat{\mu} = \overline{x} = \sum_{T} \sum_{T} \frac{96}{80} = 1.2$ M1 A1 $\Sigma fx^2 = 32 + 56 + 72 + 48 = 208$ M1 $\sigma^2 = s^2 = \frac{80}{79} (\frac{286}{80} - 1.2^2) = 1.17$ M1 A1 (c) variance ≈ mean as would be expected with a Poisson distribution B1 (d) H ₀ : Po(1.2) is a suitable model H1 H1 : Po(1.2) is not a suitable model P(1) = 1.2e^{-1.2} = 0.3614 P(2) = $\frac{12^2e^{-1.2}}{2} = 0.2169$ P(1) = 1.2e^{-1.2} = 0.3614 P(2) = $\frac{12^3e^{-1.2}}{480^2} = 0.0260$ × 80 to give exp. freqs then freq of ≥ 5 = (80 - sum of others) ∴ exp.

Performance Record – S3 Paper C

Question no.	1	2	3	4	5	6	7	8	Total
Topic(s)	sampling	dist. of sample mean	bias	linear comb. of Normal r.v.	diff. of means hyp. test	conting. table	pmcc, hyp. test	goodness of fit, Poisson	
Marks	5	6	7	7	8	11	11	20	75
Student									